Numeric Response Questions

Properties of Triangles

Q.1 In $AABC$ if $\frac{r}{(s-b)(s-c)}+\frac{r_2}{(s-c)(s-a)}+\frac{r_1}{(s-a)(s-b)}=\frac{k}{r}$, then find value of k.

Q.2 The sides of a triangle are $3+\sqrt{3}, 2\sqrt{3}$ and $\sqrt{6}$. If the least angle is k^6 then find k.

Q.3 In $\triangle ABC$, if $a=3, b=4, c=5$, then sin $2B$ is equal to $\frac{24}{k}$, then find k.

Q.4 Ratio of sides in a triangle are $3:7:8$. If circumradius (R) : inradius (r) $= k:2$ then find k.

Q.5 Find the radius of the incircle of a triangle whose sides are 18, 24 and 30 .

Q.6 In a $\triangle ABC$, if $b+c=3a$, then find the value of $\cot\frac{B}{2}\cot\frac{C}{2}$.

Q.7 In a $\triangle ABC$, if $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c}$ and the side $a=2$, then find area of the triangle.

Q.8 In a $\triangle ABC$ if $a=3, b=4, c=5$ then find its circumradius.

Q.9 In an equilateral triangle of each side $2\sqrt{3}$ cm, then find radius of the circum circle.

Q.10 If the area of a triangle is 81 and its perimeter is 27 then find its inradius.

Q.11 In $\triangle ABC$, if $a^2+b^2=671c^2$ then find $\frac{\cot C}{\cot A+\cot B}$,

Q.12 In a $\triangle ABC, a=6, b=3$ and $\cos C=\frac{\sqrt{3}}{2}$, then find area of triangle.

Q.13 In $\triangle ABC$, if $a=3, b=5, c=4$, then find value of $\sin\frac{B}{2}+\cos\frac{B}{2}$.

Q.14 In any $\triangle ABC$, find value of $\left(\frac{b-c}{a}\right)\cos^2\left(\frac{A}{2}\right)+\left(\frac{c-a}{b}\right)\cos^2\left(\frac{B}{2}\right)+\left(\frac{a-b}{2}\right)\cos^2\left(\frac{C}{2}\right)$,

Q.15 In a $\triangle ABC$, if $(b+c-a)\tan\left(\frac{A}{2}\right)$ is equal to $\frac{k\Delta}{s}$ then find k,

ANSWER KEY

1. 3.00	**2.** 30.00	**3.** 25.00	**4.** 7.00	**5.** 6.00	**6.** 2.00	**7.** 1.73
8. 2.50	**9.** 2.00	**10.** 6.00	**11.** 335.00	**12.** 4.50	**13.** 1.41	**14.** 0.00
15. 2.00						

Hints & Solutions

1. $\text{LHS} = \dfrac{\Delta}{(s-a)(s-b)(s-c)}$

$+ \dfrac{\Delta}{(s-a)(s-b)(s-c)} + \dfrac{\Delta}{(s-a)(s-b)(s-c)}$

$= \dfrac{3\Delta}{(s-a)(s-b)(s-c)}$

$= \dfrac{3s\Delta}{\Delta^2} = \dfrac{3s}{\Delta} = \dfrac{3}{r}$

2. Let least angle is A, then

$\cos A = \dfrac{(3+\sqrt{3})^2 + (2\sqrt{3})^2 - (\sqrt{6})^2}{2(3+\sqrt{3})(2\sqrt{3})}$

$= \dfrac{9+3+6\sqrt{3}+12-6}{12(\sqrt{3}+1)}$

$= \dfrac{18+6\sqrt{3}}{12(\sqrt{3}+1)} = \dfrac{\sqrt{3}}{2} = \cos 30°$

$\angle A = 30°$

3. Given a = 3, b = 4, c = 5

then $\cos B = \dfrac{3^2+5^2-4^2}{2\times3\times5} = \dfrac{9+25-16}{30}$

$= \dfrac{18}{30} = \dfrac{3}{5}$

$\Rightarrow \sin B = \dfrac{4}{5}$

therefore,

$\sin 2B = 2 \sin B \cos B = \dfrac{24}{25}$

4. Given a : b : c = 3 : 7 : 8

then $\dfrac{R}{r} = \dfrac{abc.s}{4\Delta^2}$

$= \dfrac{abc.s}{4s(s-a)(s-b)(s-c)}$

$= \dfrac{3k.7k.8k}{4(9k-3k)(9k-7k)(9k-8k)}$

$= \dfrac{3k.7k.8k}{4\times6k.2k.k} = \dfrac{7}{2}$

5. a = 18, b = 24, c = 30

$s = \dfrac{18+24+30}{2} = \dfrac{72}{2} = 36$

$r = \dfrac{\Delta}{s}$

$= \dfrac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$

$= \dfrac{\sqrt{36\times18\times12\times6}}{36}$

$= \dfrac{36\times6}{36} = 6$

6. b + c = 3a

$\Rightarrow K \sin B + K \sin C = 3 K \sin A$

$\Rightarrow 2 \sin\dfrac{B+C}{2} \cos\dfrac{B-C}{2} = 3 \cdot 2 \sin\dfrac{A}{2}\cos\dfrac{A}{2}$

$\Rightarrow \cos\dfrac{1}{2}B \cos\dfrac{1}{2}C + \sin\dfrac{1}{2}B \sin\dfrac{1}{2}C$

$$= 3\left(\cos\frac{1}{2}B\cos\frac{1}{2}C - \sin\frac{1}{2}B\sin\frac{1}{2}C\right)$$

$$\Rightarrow 2\cos\frac{1}{2}B\cos\frac{1}{2}C = 4\sin\frac{1}{2}B\sin\frac{1}{2}C$$

$$\Rightarrow \cot\frac{1}{2}B\ \cot\frac{1}{2}C = 2$$

7. $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$$\Rightarrow \frac{\cos A}{k\sin A} = \frac{\cos B}{k\sin B} = \frac{\cos C}{k\sin C}$$

$\Rightarrow \cot A = \cot B = \cot C$

$\Rightarrow A = B = C \Rightarrow$ equilateral triangle

$\therefore$ Area $= \frac{\sqrt{3}}{4}(a)^2 = \frac{\sqrt{3}}{4}(2)^2 = \sqrt{3}$

11. $\frac{\cot C}{\cot A + \cot B} = \frac{\cos C \sin A \sin B}{\sin^2 C}$

$$= \frac{a^2+b^2-c^2}{2ab} \times \frac{ab}{c^2} = \frac{670c^2}{2c^2} = 335$$

12. Given $\cos C = \frac{\sqrt{3}}{2} \Rightarrow \sin C = \frac{1}{2}$

Area of triangle $= \frac{1}{2}$ ab sin C

$$= \frac{1}{2} \times 6 \times 3 \times \frac{1}{2} = \frac{9}{2}$$

13. Given, a = 3, b = 5, c = 4

$$\sin\frac{B}{2} + \cos\frac{B}{2}$$

$$= \sqrt{\frac{(s-a)(s-c)}{ca}} + \sqrt{\frac{s(s-b)}{ca}}$$

$$= \sqrt{\frac{(6-3)(6-4)}{4\times 3}} + \sqrt{\frac{6(6-5)}{4\times 3}}$$

$$= \sqrt{\frac{3\times 2}{4\times 3}} + \sqrt{\frac{6\times 1}{4\times 3}}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$